Translations: English to SL

Complex English sentences are composed of

- simpler English sentences
- logical connectives of English

When translating English into SL

- the simplest English sentences are translated into *sentence letters* of SL
- the logical connectives of English are translated into the logical connectives of SL: $\sim, \wedge, \vee, \rightarrow, \leftrightarrow$

But what are the logical connectives of English? How to tell when an English sentence is simple or complex?

Translations: English to SL

But what are the logical connectives of English? How to tell when an English sentence is simple or complex?

Here's a poor candidate for a simple sentence

1. Mares eat oats and does eat oats and little lambs eat ivy.

- Here we have three sentences connected into a larger sentence:
  1. Mares eat oats.
  2. Does eat oats.
  3. Little lambs eat ivy.
- What can we say about the English-language connective 'and'?

Translations: English to SL

All connectives of SL are *truth-functional* connectives:

- they're *connectives* because they connect one or more sentences together to form a new sentence:
  - A becomes $\sim A$
  - A together with B can become $A \wedge B$, $A \rightarrow B$, ...
- They're *truth-functional* because the truth value of the higher-order sentence is a function of the truth value of its proper subsentences
  - truth value of $A \wedge B$ is a function of the truth value of A and of B

Translations: English to SL

Not all English connectives are truth functional:

- examples:
  - Grass is green *because* snow is white.
  - Dogs bark *entails that* Cats jump *entails that* memes bore.
  - *Luckily*, Nate got the job.
  - *Necessarily,* 2+2=4.
  - *Probably,* everything went fine.
- Each is a *connective* because it connects one or more sentence to form a new sentence
- But none is *truth functional*, since the truth value of the full sentence isn't a function of the truth values of the non-italicized bits

Translations: English to SL

English's 'and' and SL's '$\wedge$' express the same truth function

- the new sentences formed by 'and' and by '$\wedge$' share the same truth value whenever the constituent sentences the made them up share the same truth values.

Example:

4. The sun is shining and the birds are chirping.

is true iff the following two sentences are also true:

5. The sun is shining.
6. The birds are chirping.

Translations: English to SL

4. The sun is shining <u>and</u> the birds are chirping.

Translation Key:

A: The sun is shining.

O: The birds are chirping.

So (4.) is translated into the SL: $(A \wedge O)$

Translations: English to SL

Sometimes the logical structure of English sentences isn't so obvious:

7. Nathanael Green *and* 'Light Horse Harry' Lee were officers in the Continental Army.

But (7.) is true iff (8.) and (9.) are true:

8. Nathanael Green was an officer in the Continental Army.
9. 'Light Horse Harry' Lee was an officer in the Continental Army.

So (7.)'s structure is more perspicuous when expressed by the equivalent:

7.' Nathanael Green was an officer in the Continental Army *and* Light Horse Harry' Lee was an officer in the Continental Army.

Translations: English to SL

7. Nathanael Green *and* 'Light Horse Harry' Lee were officers in the Continental Army.

Translation Key:

B: Nathanael Green was an officer in the Continental Army.

C: 'Light Horse Harry' Lee was an officer in the Continental Army.

So (7.) is translated into the SL: $(B \wedge C)$

Translations: English to SL

Sometimes English words other than ‘and’ express the same truth function:

10. Mary wants to drive her mother’s car, *but* she is *not* old enough.

Simple sentences in (10.):

(i) Mary wants to drive her mother’s car

(ii) Mary is old enough to drive her mother’s car

Translations: English to SL

Sometimes English words other than ‘and’ express the same truth function:

10. Mary wants to drive her mother’s car, *but* she is *not* old enough.

Simple sentences in (10.):

(i) Mary wants to drive her mother’s car

(i) Mary is old enough to drive her mother’s car

(10.) is true iff (i) is true and (ii) is false

Translation Key:

A: Mary wants to drive her mother’s car.

B: Mary is old enough to drive her mother’s car.

So (10.) is translated into the SL: $(A \wedge {\sim}B)$

Translations: English to SL

Consider:

10. Mary wants to drive her mother's car, *but* she is *not* old enough.

and

10*. Mary wants to drive her mother's car, *and* she is *not* old enough.

Given our translation key, both English sentences are translated into SL as: $(A \wedge \sim B)$

Do (10.) and (10*.) mean the same thing?

Translations: English to SL

Similar example:

12. John took off his clothes and went to bed.
13. John went to bed and took off his clothes

SL translations of (12.) and (13.) will be TFE.

Should they be?

Translations: English to SL

Grandy's take: that's fine, since neither sentence literally asserts an order in which the sentences occur.

Test for whether sentence Φ literally asserts Θ:

- if 'Φ and not Θ' implies a *contradiction*, then Φ literally asserts that Θ
- if 'Φ and not Θ' *does not* imply a *contradiction*, then Φ does not literally assert that Θ

Contrast:

12*. John took off his clothes and went to bed, but possibly not in that order.

12.** John took off his clothes and went to bed, but he didn't take off his clothes.

Translations: English to SL

Translating ‘or’:

15. *Either* the tigers will get us *or* the lions will.

Simple sentences:

(i) The tigers will get us.

(ii) The lions will get us.

Arguably (15.) is true iff (i) is true, or (ii) is true, or both.

Translations: English to SL

Translating 'or':

15. *Either* the tigers will get us *or* the lions will.

Translation Key:

I: The tigers will get us.

L: The lions will get us.

So, (15.) is translated into SL as: $(I \vee L)$

Translations: English to SL

BUT, consider:

16. Tom may have the soup or Tom may have the salad.

When (16.) is uttered by the waiter at a fancy restaurant, how many choices does Tom have?

1. the soup
2. the salad
3. both(?)

Translations: English to SL

Two readings of the English 'or':

Inclusive 'or': 'Φ or Θ' is true iff Φ is true, or Θ is true, or both

Exclusive 'or': 'Φ or Θ' is true iff Φ is true, or Θ is true, but false if both Φ and Θ are true

Translations: English to SL

16. Tom may have the soup or Tom may have the salad.

How to translate (16.) into SL?

Translation Key:

A: Tom may have the soup.

B: Tom may have the salad.

Inclusive reading of ‘or’:

- $(A \vee B)$

Exclusive reading of ‘or’:

- $((A \vee B) \wedge \sim(A \wedge B))$

Translations: English to SL

Two readings of the English 'or':

Inclusive 'or': 'Φ or Θ' is true iff Φ is true, or Θ is true, or both

Exclusive 'or': 'Φ or Θ' is true iff Φ is true, or Θ is true, but false if both Φ and Θ are true

Which is the correct reading of the English 'or'?

Translations: English to SL

Translating conditionals:

17. *If* George Washington crosses the Delaware River *then* the Hessians will be defeated.

Translation Key:

A: George Washington crosses the Delaware River.
B: The Hessians will be defeated.

But: is ‘$(A \to B)$’ an adequate translation of (17.)?

- SL sentences of the form ‘$(\phi \to \theta)$’ are false when the LHS is true and the RHS is false; otherwise they're true.
- Does the English ‘if...then...’ work that way as well?

Translations: English to SL

Consider English language conditionals where both sides are true:

17. *If* George Washington crosses the Delaware River *then* the Hessians will be defeated.

17*. *if* Rice is in Houston *then* Rice is in Texas.

17.** *if* x>y *then* y<x

- Each English conditional has true sentences on both LHS and RHS
- Each English conditional seems true
- SL sentences of the form '$(\phi \to \theta)$' with true LHS and RHS sentences are true
- So far, '$\to$' seems like a good translation of the English 'if...then...'

Translations: English to SL

Consider English language conditionals where LHS is true and RHS is false:

(i) If George Washington crossed the Delaware, then George Washington is an otter.

(ii) If Rice University is in Houston, then Houston is in Oklahoma.

Suppose (i) and (ii) each translate to an SL sentence of the form '$(\phi \rightarrow \theta)$'

- then both (i) and (ii) are false, since the LHS is true and the RHS is false
- Ignoring SL for a moment: in your judgment as a competent speaker of English, are (i) and (ii) both false?

Translations: English to SL

Consider English language conditionals where both sides are false:

(i) If George Washington landed on the moon, then George Washington landed on the moon.

(ii) If George Washington landed on the moon, then Hillary Clinton is president.

Suppose (i) and (ii) each translate to a SL sentence of the form '$(\phi \to \theta)$'

- then both (i) and (ii) are true, since the LHS is false
- Ignoring SL for a moment: in your judgment as a competent speaker of English, are (i) and (ii) both true?
- lesson?

Translations: English to SL

Consider English language conditionals where the LHS is false and the RHS is true

(i) If George Washington landed on the moon, then George Washington crossed the Delaware river.

Suppose (i) translates to an SL sentence of the form '$(\phi \rightarrow \theta)$'

- then (i) is true, since the LHS is false
- Ignoring SL for a moment: in your judgment as a competent speaker of English, is (i) true?

Translations: English to SL

Consider English language conditionals where the LHS is false and the RHS is true

Now consider:

(i*) If George Washington crossed the Delaware river and Houston is in Oklahoma, then George Washington crossed the Delaware river.

Translation Key:

A: George Washington crossed the Delaware River.
B: Houston is in Oklahoma

- If 'if...then' translates to '$\rightarrow$', then (i*) becomes $((A \wedge B) \rightarrow A)$
- B is false, so $(A \wedge B)$ is false, so $((A \wedge B) \rightarrow A)$ is true
- does that match your intuition about the truth of (i*)?

Translations: English to SL

So: does the English ‘if...then’ translate to the SL ‘$\rightarrow$’?

Translations: English to SL

Scope of connectives

Recall:

- one connective is within the *scope* of another iff the first connective is within a sentence directly connected with the second connective
- the *main connective* is the one connective of the sentence that's not within the scope of any other connectives

Translations: English to SL

Scope of connectives

21. Either Mia flew *or* both Jackson *and* Harper took off early.

What's the main connective?

Translations: English to SL

Scope of connectives

21. Either Mia flew or both Jackson and Harper took off early.

becomes...

22. (Mia flew) $\vee$ (both Jackson and Harper took off early).

becomes...

23. (Mia flew) $\vee$ ((Jackson took off early) $\wedge$ (Harper took off early)).

Translation Key:

M: Mia flew.

J: Jackson took off early.

H: Harper took off early.

becomes...

24. $M \vee (J \wedge H)$

Translations: English to SL

Scope of connectives

But consider the very similar (21*.):

21*. Mia played or Jackson played and Harper played

Two possible readings:

21*. Mia played or (Jackson played and Harper played).

21.** (Mia played or Jackson played) and Harper played.

Translation Key:

M: Mia played.

J: Jackson played.

H: Harper played.

In SL:

- (21*.) $(M \vee (J \wedge H))$
- (21**.) $((M \vee J) \wedge H)$

Very different sentences! Lesson?

Translations: English to SL

Scope of connectives

25. *If* Mia got the job *and* Jackson *didn't*, *then* Mia will take off tomorrow *and* Harper will have to come in.

Identifying the main connective we get:

26. (Mia got the job *and* Jackson *didn't*) $\rightarrow$ (Mia will take off tomorrow *and* Harper will have to come in).

becomes:

27. ((Mia got the job) $\wedge$ (Jackson *didn't* get the job)) $\rightarrow$ (Mia will take off tomorrow *and* Harper will have to come in).

becomes:

28. ((Mia got the job) $\wedge$ ($\sim$ (Jackson got the job))) $\rightarrow$ ((Mia will take off tomorrow) $\wedge$ (Harper will have to come in)).

Translations: English to SL

Scope of connectives

25. *If* Mia got the job *and* Jackson *didn't*, *then* Mia will take off tomorrow *and* Harper will have to come in.

...

28. ((Mia got the job) $\wedge$ ($\sim$ (Jackson got the job))) $\rightarrow$ ((Mia will take off tomorrow) $\wedge$ (Harper will have to come in)).

Translation Key:

M: Mia got the job.

J: Jackson got the job.

I: Mia will take off tomorrow.

H: Harper will have to come in late.

becomes:

29. $(M \wedge {\sim}J) \rightarrow (I \wedge H)$

Translations: English to QL

1. Mary is happy, smart, adorable, and a child.

- No truth functional connectives, so no logical structure representable in SL:

Translation Key:

H: Mary is happy.

R: Mary is smart.

A: Mary is adorable.

C: Mary is a child.

- SL translation: $(H \wedge R \wedge A \wedge C)$
- that's not great

Translations: English to QL

1. Mary is happy, smart, adorable, and a child.

- Even without quantifiers, QL does much better:

Translation Key:

m: Mary

Ht: t happy.

Rt: t is smart.

At: t is adorable.

Ct: t is a child.

- QL translation: $(H\mathrm{m} \wedge R\mathrm{m} \wedge A\mathrm{m} \wedge C\mathrm{m})$.
- much better

Translations: English to QL

Translating universal quantifiers:

3. All dogs are furry.

- (3.) asserts a *subset* relation:
  - the set of all dogs is a subset of all furry things
  - In set-theory notation: $\{x|dog(x)\} \subseteq \{y|furry(y)\}$
- Standard way to express $\phi \subseteq \theta$ in QL:
  - $\forall\alpha(\phi \rightarrow \theta)$

Translations: English to QL

Translating universal quantifiers:

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- Standard way to express $\phi \subseteq \theta$ in QL:
  - $\forall\alpha(\phi \rightarrow \theta)$

Animals model:

$Animals(U)$: All animals.
$Animals(D^{'})$: is a dog.
$Animals(R^{'})$: is furry.

- Translation of (3.) into QL: $\forall x(Dx \rightarrow Rx)$

Translations: English to QL

More examples:

Animals model:

$Animals(U)$: All animals.
$Animals(D')$: is a dog.
$Animals(R')$: is furry.

4. No dogs are furry.

translation: $\forall x(Dx \rightarrow {\sim}Rx)$

Translations: English to QL

More examples:

Animals model:

$Animals(U)$: All animals.
$Animals(D')$: is a dog.
$Animals(R')$: is furry.

5. Only dogs are furry.

translation: $\forall x(Rx \to Dx)$

Translations: English to QL

More examples:

Animals model:

$Animals(U)$: All animals.
$Animals(D^{'})$: is a dog.
$Animals(R^{'})$: is furry.

6. Some dogs are furry

translation: $\exists x(Dx \wedge Rx)$

Question: how many furry dogs must there be in order for (6.) to be true?

Translations: English to QL

More examples:

Animals model:

$Animals(U)$: All animals.
$Animals(D')$: is a dog.
$Animals(E')$: is energetic.
$Animals(H')$: is a happy.
$Animals(R')$: is furry.

8. All happy dogs are furry and energetic

translation: $\forall x((Hx \wedge Dx) \rightarrow (Rx \wedge Ex))$

Translations: English to QL

More examples:

Animals model:

$Animals(U)$: All animals.
$Animals(A')$: is a mammal.
$Animals(C')$: is a cat.
$Animals(D')$: is a dog.

9. All cats and dogs are mammals.

bad translation: $\forall x((Cx \wedge Dx) \rightarrow Ax)$

translation: $\forall x((Cx \vee Dx) \rightarrow Ax)$

Translations: English to QL

More examples:

Animals model:

$Animals(U)$: All animals.
$Animals(C')$: is a cat.
$Animals(D')$: is a dog.
$Animals(A'')$: is smarter than.

10. All dogs are smarter than all cats.

translation: $\forall x(Dx \rightarrow \forall y(Cy \rightarrow Axy))$

Practice translations

	Symbol	Model Assignment
Universe:		The set of states
Constants:	m	MT
1 place predicates:	P'	Pacific states
	A'	Atlantic states
	C'	Coastal states
2 place predicates:	B''	borders

1. All Atlantic states are coastal.
2. No Atlantic state is Pacific.
3. Every state borders some state.
4. Every state does not border Montana.

Universe:		The set of states
1 place predicates:	P'	Pacific states
	A'	Atlantic states
	G'	Gulf states
	M'	Mountainous states
	C'	Coastal states
2 place predicates:	L''	is larger than (area)
2 place predicates:	B''	borders

1. Some Gulf state is larger than all states that border it.
2. All Atlantic states are coastal.
3. No Atlantic state is Pacific.
4. Every state is larger than some state.
5. Every state is not larger than Montana.
6. Some Gulf state is an Atlantic state.
7. All states that border a Pacific state are mountainous.
8. Any state that is mountainous is larger than Rhode Island.
9. Any state that is mountainous is larger than all Atlantic states.
10. If any state is mountainous, California is.
11. If any state is mountainous, it is larger than Rhode Island.
12. Any state that has no bordering states is mountainous.
13. All states that are bigger than all mountainous states are coastal.
14. No state is bigger than Montana unless it is coastal.